Simplified Determination of Impedances of Chebyshev Transformers

If it is desired to match two different impedances by means of cascaded quarter-wavelength transformers, one may possibly refer to the tables of Young [1] or to the design curves of Jasik [2]. Because these works limit their information to less than seven sections, it is not possible to easily obtain the characteristic impedances for transformers which must operate over a large frequency band or which have very low reflection coefficients. The evaluation of the characteristic impedances for transformers having eight or more sections is very tedious. One can, however, evolve a relationship between the characteristic impedances of Chebyshev transformers and the excitation coefficients of Chebyshev arrays.

A design constant "C" has been defined by Jasik and is a measure of the difficulty of transformation. This design constant may be expressed as a function of the impedance transformation ratio and the maximum standing wave ratio, or as a function of the number of transformer sections and the ratio of the maximum to minimum operating frequencies. Knowing or assuming three of the above parameters, the fourth can be determined. Generally the unknown parameter is the number of transformer sections, and it can be evaluated from the expression involving the design constant C.

By comparing the transformer theory with antenna array theory it can be shown that the number of quarter-wavelength sections N corresponds to one less than the number of elements in an array. The design constant C corresponds to the voltage sidelobe level of an array.

Once the number of elements and the sidelobe level of the equivalent Chebyshev array are determined, then available tables [3] or graphs [4] may be used to determine the excitation coefficients. These excitation coefficients correspond to the a_m's as derived by Cohn [5]. The ratio of impedances at each step is Z_{m+1}/Z_m.

A relationship between the characteristic impedances of each section of the transformer, the input and terminating impedances, and the a_m's are given by the expression

$$\ln \frac{Z_{m+1}}{Z_m} = \frac{a_m \ln \frac{Z_{n+1}}{Z_1}}{\sum_1^n a_m}$$

where

$$n = N + 1.$$

Once the number of transformer sections and the design constant C have been calculated, the array which most closely satisfies these parameters may be chosen. The maximum VSWR and bandwidth may be recalculated from these data. In this manner the total time required to calculate the impedances of a multisection transformer can be minimized.

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References

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